GeoCa 23

Horní Lysečiny "Integrál"

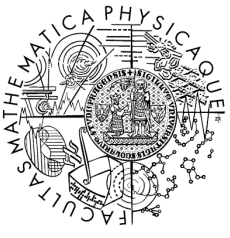
6-10 December 2023

Welcome to the 3rd GeoCa conference in Horní Lysečiny

The Department of Mathematical analysis of Charles University welcomes you to our workshop GeoCa 23 on Geometric Analysis and Calculus of Variations in Lysečiny. According to our aim, we have brought together leading minds in the field (complete with their bodies) and young researchers in a relaxed, informal environment conducive of forging new scientific colaborations.

It is the sincere wish of the organising committee that you enjoy your time.

Daniel



FACULTY
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Abstracts

Thin film asymptotics in a model related to single-slip crystal plasticity

Thurs 1145

Stefan Krömer

ÚTIA, Czech Academy of Sciences, Prague, Czech Republic

We perform a 2d-1d dimension reduction analysis of a model featuring strong local constraints: incompressibility and inextensibility in a particular direction. The model can be interpreted to describe either the first discrete time step of the evolution of a single-slip elastoplastic body in large deformations with fully rigid elasticity, or, alternatively, the static equilibrium of a fibre-reinforced incompressible elastic body.

We show that in the thin-film limit, deformations can essentially freely bend despite the restrictive constraints, exploiting suitable nonsmooth deformations. We also prove this nonsmoothness of almost optimal configurations is crucial in general; in fact, an energy gap (a Lavrentiev phenomenon) occurs if we artificially restrict our model to smooth deformations. The latter phenomenon is absent if the differential constraints are appropriately softened.

Joint work: Dominik Engl (Eichstätt), Martin Kružík (Prague).

Behind the Regularity: local and nonlocal models with energy gaps

Thurs 1330

Anna Balci

Bielefeld University

In this talk I discuss resent results on scalar problems with Lavrentiev gap, connected questions on irregularity of scalar minimizers and nondensity of smooth function in the corresponding energy function spaces for different models including variational problems with differential forms and nonlocal problems. We design new examples using fractal Cantor barrier sets and present numerical scheme that converges to the global minimizer.

Inertial (self-)collisions of viscoelastic solids with Lipschitz boundaries

Thurs 1415

Antonín Česík
Charles University, Prague

We continue our study, started in a previous paper, of (self-)collisions of viscoelastic solids in an inertial regime. We show existence of weak solutions with a corresponding contact force measure in the case of solids with only Lipschitz-regular boundaries. This necessitates a careful study of different concepts of tangent and normal cones and the role these play both in the proofs and in the formulation of the problem itself. Consistent with our previous approach, we study contact without resorting to penalization, i.e. by only relying on a strict non-interpenetration condition. This is a joint work with G. Gravina and M. Kampschulte.

Squashing measures on purely unrectifiable spaces and spaces admitting weak tangent fields

Thurs 1530

Jakub Takáč
University of Warwick

Consider a purely unrectifiable set $S \subset \mathbb{R}^2$ (i.e. the intersection of S with every rectifiable curve has length 0). Good examples to keep in mind are the four corner Cantor set (of finite, positive $\mathcal{H}^1$ measure) and the Koch curve (of non σ-finite $\mathcal{H}^1$ measure). Using the Besicovitch-Federer projection theorem, it is essentially an easy exercise to show that if $\mathcal{H}^1(S) < \infty$ and S is compact, there exists a sequence $f_n \colon \mathbb{R}^2 \to \mathbb{R}^2$ of 1-Lipschitz maps such that

1. $||f_n - \mathrm{Id}\,||_\infty \to 0$,
2. $f_n(S)$ is a *finite* set.

The natural question is whether $\mathcal{H}^1(S) < \infty$ is an assumption that can be dropped. The answer is obviously negative: if S is connected and non-trivial (e.g. the Koch curve) then for any $f \colon \mathbb{R}^2 \to \mathbb{R}^2$ continuous and sufficiently close to identity, $f(S)$ is connected and non-trivial, hence infinite (even positive $\mathcal{H}^1$ measure). Instead, we prove that if S is compact and purely unrectifiable, then for *any* Borel measure μ supported on S, there exists a sequence $f_n \colon \mathbb{R}^2 \to \mathbb{R}^2$ of 1-Lipschitz maps such that

1. $||f_n - \mathrm{Id}\,||_\infty \to 0$,

2. $f_{n\#}\mu$ is a 0-*dimensional* measure.

We discuss this result in the setting of metric spaces and also its higher dimensional variants, where pure unrectifiability needs to be replaced with an appropriate notion, that of a weak tangent field. This is joint work with Giovanni Alberti, David Bate and Andrea Marchese.

Fractional Orlicz-Sobolev spaces and modulus of continuity

Thurs 1615

Luboš Pick

Charles University, Prague

We build a fractional Sobolev space upon an Orlicz space using an appropriate version of the Gagliardo-Slobodeckij functional. We study the question under which conditions upon the Young function A, which generates the underlying Orlicz space, do all the functions from the corresponding Orlicz-Sobolev space $W^{m,A}(\mathbb{R}^n)$ enjoy a universal modulus of continuity. We present optimal embeddings of Orlicz-Sobolev spaces into the spaces of uniformly continuous functions enjoying a universal modulus of continuity. We discover some dissimilarities in comparison to the classical Sobolev spaces, in particular, we point out that the factorization through Campanato spaces, which effectively works in the classical case, is for Orlicz-Sobolev spaces useless as it leads to non-sharp results.

Sobolev-spaces on changing domains

Fri 1145

Malte Kampschulte

Charles University, Prague

In many parts of modern continuum mechanics such as fluid structure interaction, the shape of the domain itself can become part of the problem. For the analysis of such problems one thus needs to ask the question: What does it mean for a sequence of functions defined on different domains to converge to a function on a limit domain. The aim of this talk is to give a short overview over the state of the art in such problems and then to introduce our own minimalist approach to strong and weak convergence using zero-extensions. In contrast to the usual methods this approach does not rely on any parametrization of the domain. Yet under only some mild assumptions on the convergence and regularity of the domains, we are able to

derive counterparts of classic results in the fixed domain case, including crucial compactness-properties. This is a joint work in progress with N.Evseev and A.Menovshikov.

Rigidity and large volume residues in exterior isoperimetry for convex sets

Friday 1330

Nicola Fusco

Università degli Studi di Napoli "Federico II"

A comparison theorem by Choe, Ghomi and Ritoré states that the exterior isoperimetric profile I_C of any convex body C in R^N lies above that of any half-space H. We characterize convex bodies such that $I_C \equiv I_H$ in terms of a notion of "maximal affine dimension at infinity", briefly called the asymptotic dimension $d^*(C)$ of C. More precisely, we show that $I_C \equiv I_H$ if and only if $d^*(C) \geq N-1$. We also show that if $d^*(C) \leq N-2$, then, for large volumes v, I_C is asymptotic to the isoperimetric profile of R^N. We then estimate, in terms of $d^*(C)$-dependent power laws, the order as $v \to \infty$ of the difference between I_C and the isoperimetric profile of R^N.

Differentiability of weak limits of bi-Sobolev homeomorphisms

Fri 1415

Anastasia Molchanova

University of Vienna

This talk discusses the differentiability of weak limits of bi-Sobolev homeomorphisms. Given $p > n-1$, consider a sequence of homeomorphisms f_k with positive Jacobians $J_{f_k} > 0$ almost everywhere and finite energy $\sup(||f_k||_{1,n-1} + ||f^{-1}||_{1,p}) < \infty$. Then the functions f and h, the weak limits of f_k and f_k^{-1} respectively, with positive Jacobians $J_f > 0$ and $J_h > 0$ a.e., are inverse, in the sense that $h(f(x)) = x$ a.e. and $f(h(y)) = y$ a.e., and f and h are differentiable almost everywhere.

This is a joint work with Anna Doležalová.

Regularity results for Legendre-Hadamard elliptic systems

Fri 1530

Christopher Irving
Technische Universität Dortmund

I will discuss the regularity of solutions to elliptic systems under certain a-priori assumptions. While it is known that Legendre-Hadamard ellipticity is insufficient for even partial C^1 regularity, we will show that such counterexamples may be ruled out by imposing a smallness condition of the gradient in BMO. Focusing on critical points of Euler-Lagrange systems, global regularity results will be presented, along with some ideas behind the proof.

The saga of estimation of intermediate derivative

Fri 1615

Filip Soudský
Technical University Liberec

We point out the main milestones in the estimation of the intermediate derivative and show some of its applications. We also show some of the pointwise estimates and their limitations. We will introduce a new type of pointwise estimates to help us gain a general form of Gagliardo-Nirenberg inequality for arbitrary rearrangement-invariant Banach function spaces. We also show that this result is optimal in the class of rearrangement-invariant spaces.

Weak limits of Sobolev homeomorphisms and injectivity a.e.

Sat 1145

Stanislav Hencl
Charles University, Prague

One of the crutial properties in models of Nonlinear Elasticity is the injectivity of the deformation as it corresponds to the interpenetration of the matter. We study when the mappings in the class of weak limits of Sobolev homeomorphisms with $J_f > 0$ a.e. are injective a.e. This is a joint result with O. Bouchala and Z. Zheng.

The nonlinear variant of Gagliardo-Nirenberg inequality and application to PDE

Sat
1330

Tomáš Roskovec

University of South Bohemia

We introduce the new version of Gagliardo-Nirenberg inequality, the estimate of the norm of intermediate derivative by the term based on the higher derivative and the function itself. Instead of the classical product of powers of norms, we use different functions. The estimate is shown to be applied to elliptic PDEs with nonlinear coefficients. A joint work with Agnieszka Kalamajska and Dalimil Peša.

Quantitative estimates for norm convergence of multiple ergodic averages

Sat
1415

Lenka Slavíková

Charles University, Prague

Multiple ergodic averages arose from the work of Fürstenberg and his coauthors, dating back to the 1970s, which connected ergodic theory with arithmetic combinatorics. These averages are associated with m commuting measure preserving transformations $T_1, \dots, T_m$ on a σ-finite measure space X, and they have the form

$$M_N(f_1, \dots, f_m)(x) = \frac{1}{N} \sum_{i=0}^{N-1} f_1(T_1^i x) \dots f_m(T_m^i x).$$

It is known that these averages converge in the $L^2(X)$-norm (Tao, 2008), but quantitative estimates for the rate of the convergence are not yet well understood. In this talk, we focus on such quantitative estimates, and after reviewing known bounds for single and double ergodic averages, we establish a new quantitative estimate for the norm convergence of triple ergodic averages. This is a joint work with Polona Durcik and Christoph Thiele.

Improved integrability of gradients of quasiminima in metric spaces

Sat 1530

Antonella Nastasi
University of Palermo

We study local and global higher integrability for upper gradients of quasiminima of a double phase Dirichlet integral with (p, q)-growth with fixed boundary data, assuming it belongs to a slightly better Newtonian space.

The setting is the general context of metric measure spaces, endowed with a doubling metric measure and supporting a Poincaré inequality.

The proofs follow a variational approach and they are based on the De Giorgi method, a careful phase analysis and estimates in the intrinsic geometries.

This is a joint work [2] with Juha Kinnunen (Aalto University) and Cintia Pacchiano Camacho (University of Calgary).

[1] C. De Filippis, G. Mingione, *Manifold constrained non-uniformly elliptic problems*, J. Geom. Anal., 30 (2020), 1661–1723.

[2] J. Kinnunen, A. Nastasi, C. Pacchiano Camacho, *Gradient higher integrability for double phase problems on metric measure spaces*, Proc. Amer. Math. Soc. (to appear), (2023).

[3] O. E. Maasalo and A. Zatorska-Goldstein, *Stability of quasiminimizers of the p-Dirichlet integral with varying p on metric spaces*, J. London Math. Soc. (2) 77 (2008), 771–788.

Stability for anisotropic curvature functionals

Sat 1615

Julian Scheuer
Goethe Universität Frankfurt

Anisotropic integrands are used to model surface energies of inhomogeneous materials, where the energy density depends on the direction of the surface. Under suitable assumptions, minimisers of the associated functionals are known to be so-called Wulff shapes, those being certain convex bodies which induce a Minkowski norm on Euclidean space. The first variation being the anisotropic mean curvature, the anisotropic Alexandrov-type theorem says that a surface with constant anisotropic mean curvature must be the corresponding Wulff shape. There are various related rigidity results for other anisotropic curvature functionals and in this talk we discuss their stability: In case the curvature condition is "almost" satisfied, is the surface "close" to the Wulff shape? This is joint work with Xuwen Zhang.

Author Index

Your notes

. . . the making of many books hath no end, and much study is a weariness of the flesh. . .

Ecclesiates 12:12

. . . it is the spirit in a person, the breath of the Almighty, that gives them understanding.

Job 32:8

Pro tohle se elektrárny stavět nebudou!

Honza Malý

Thank you for attending